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Regular non-twisting S-branes

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Abstract: We construct a family of time and angular dependent, regular S-brane solutions which corresponds to a simple analytical continuation of the Zipoy-Voorhees 4-dimensional vacuum spacetime. The solutions are asymptotically flat and turn out to be free of singularities without requiring a twist in space. They can be considered as the simplest non-singular generalization of the singular S0-brane solution. We analyze the properties of a representative of this family of solutions and show that it resembles to some extent the asymptotic properties of the regular Kerr S-brane. The R-symmetry corresponds, however, to the general lorentzian symmetry. Several generalizations of this regular solution are derived which include a charged S-brane and an additional dilatonic field.

Keywords: Superstrings and Heterotic Strings, p-branes.

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Contents

1. introduction

Spacelike branes, or S-branes, are spacelike surfaces similar to ordinary branes with the special characteristic that one of its transverse dimensions includes time. They can be interpreted as soliton-like, time dependent field configurations. In string theory, the study of the potential for the open string tachyon field [1] and the search for solutions describing cosmological scenarios [2] have led to the introduction of S-branes. Their study has received special attention over the past few years, motivated principally by interest in understanding the dynamics of time dependent backgrounds in string theory. In particular, S-branes can be considered as describing the formation and decay of unstable branes. From the physical point of view, the process associated with the formation and decay of a brane is expected to be smooth. In the supergravity approximation, however, it turns out that S-branes solutions are plagued by singularities [1, 3], raising the question of whether these singularities appear as a result of the supergravity approximation or due to other reasons. Many other time dependent, asymptotically flat S-branes solutions have been analyzed in several works [3, 4], but they contain either null singularities or naked timelike singularities inside internal static regions. Recently, it has been proved that this singular behavior is

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an intrinsic property of a large class of solutions [5]. The question arose as to whether, in general, there exist regular S-brane solutions. This problem is known as the singularity problem of S-branes.

Among the alternatives suggested to solve the singularity problem [1], the reduction of the R-symmetry of the S-branes represents an interesting approach. The idea is that by reducing the R-symmetry, which represents the symmetry transverse to the S-brane worldvolume, the S-brane could be localized in space and time. In fact, this procedure has been performed recently in [6] and [7], in the framework of the low-energy limit of string theory, where it was shown that regular solutions exist which are less symmetric than the S-brane solutions already discussed in the literature. The new regular solution has been obtained by applying the method of analytical continuation to the Kerr spacetime and it has been interpreted as a twisting S-brane [6]. The "rotational" part of the Kerr geometry transforms into a twist in space and the Kerr angular momentum is reinterpreted as a twist parameter that determines the global properties of the S-brane solution. In the limiting case where the Kerr angular parameter vanishes, it contains as a special case the singular S0-brane. Otherwise, the Kerr S-brane solution is regular on the entire manifold and can also be generalized to include the case of higher dimensions [7, 8]. One could then imagine that the twist in space is a necessary condition in order to get rid of the singularity. We will see that this is not necessarily true. In fact, in a recent work [9] an analytical continuation of an array of Reissner-Nordstrom black holes has been interpreted as a regular S-brane configuration. In a different approach [10], regular solutions have been found in an analytical continuation of an AdS black hole.

In this work, we present a family of regular, non-twisting S-brane solutions which we derive from the static axisymmetric Zipoy-Voorhees spacetime by applying the method of analytical continuation. In a previous work [11], we introduced the horizon method as a procedure for generating Gowdy cosmological models in General Relativity. We have shown [12] that the Kerr geometry inside the horizons can be interpreted as a Gowdy cosmology with topology $S^1 \times S^2$. More recently [13], we proposed that the cosmological configurations, or ordinary D-branes, obtained by the horizon method can be used to generate iD-branes which correspond to S-branes. In the present work we generate a special type of S-brane solution by applying the method of analytical continuation (which would correspond to the i-horizon method, in our terminology). In fact, we will see that instead of performing the analytical continuation of the Schwarzschild geometry "outside the horizon", one can also perform a similar transformation "inside the horizon" and this is sufficient to avoid the timelike naked singularity present in the non-regular S0-brane solution.

All the solutions we present in this work are asymptotically flat and can be classified by a real parameter that determines the explicit time dependence of the corresponding metric and curvature of the S-branes. In general, these solutions are simpler than previous regular S-brane solutions known in the literature and can be interpreted as the simplest regular generalizations of the singular S0-brane solution. We analyze the main properties of the simplest representative of this family of regular solutions. We show that the regular S0-brane inherits all the symmetry properties of the original Zipoy-Voorhees solution, but

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their physical interpretation is quite different. In particular, we will see that the i-rotation eliminates the original singularity and that a free parameter entering the metric can even be used to eliminate the only existent Killing horizon. But if we insist on preserving the Killing horizon, no changes in the signature of the metric occur when crossing the horizon, i.e. all the coordinates are well behaved on both sides of the horizon. In this case, the near horizon limit of the regular S0-brane solution can be shown to be described by a de Sitter space. We analyze the R-symmetry of the solution and show that it corresponds to the general lorentzian symmetry of a 2-dimensional conformally flat euclidean space. In the analysis of the asymptotic behavior of the solution we find that the spatial asymptote corresponds to a Rindler space with an exponential expansion in the angular direction, a behavior that coincides with that of the regular Kerr S-brane. For the temporal asymptote we find that the spacetime transverse to the worldvolume of the brane corresponds to a 2-dimensional Minkowski spacetime with an exponential expansion in the additional angular direction. We also derive the charged generalization of the regular S0-brane solution and generate a solution which contains the additional dilatonic field that arises in the low-energy limit of the IIA string theory. We show that the dilatonic field modifies the asymptotic temporal behavior of the regular S0-brane solution and reduces its lorentzian R-symmetry to an SO(2) symmetry.

The paper is organized as follows. After briefly reviewing the main properties of the 4-dimensional Zipoy-Voorhees spacetime in section 2.1, in section 2.2 we use the Schwarzschild metric in the Zipoy-Voorhees form "inside the horizon" to derive the simplest regular non-twisting S-brane solution which we will call the *regular S0-brane solution*. To this end we apply the method of analytical continuation that has been used in previous works to generate S-brane solutions. In section 3 we study the main global properties of this solution. Then, in section 4, we derive different generalized solutions of the regular S0-brane solution. In particular, we generate and briefly analyze a solution that includes, in addition to an electric charge monopole, a dilatonic field. In section 5 we present a family of 4-dimensional, non-twisting regular S-brane solutions which depend on two real parameters, δ and μ, and reduces to the special case analyzed in section 2.2 when both parameters coincide and are taken as $\delta = \mu = 1$. Finally, in section 6 we comment on our results and discuss the possibility of generalizing our results to include the case of higher dimensions and additional fields of interest in string theory.

2. A regular non-twisting S-brane

This section is devoted to the construction and discussion of the main properties of the simplest, non-twisting, singularity-free S-brane solution. We first present a brief review of the Zipoy-Voorhees spacetime which is described by a static, axially symmetric solution of the Einstein vacuum field equations and contains the Schwarzschild solution as a special case. Then we consider the special Schwarzschild solution "inside the horizon" and apply the method of analytical continuation to derive the corresponding S-brane solution, under the condition that it is regular on the entire manifold.

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2.1 The Zipoy-Voorhees spacetime

The Zipoy-Voorhees [14] metric in Lewis-Papapetrou form and prolate spheroidal coordinates (t, x, y, φ) has the form

$$ds^2 = -f dt^2 + \sigma^2 f^{-1}(x^2-1)(1-y^2)d\varphi^2 + \sigma^2 f^{-1} e^{2\gamma}(x^2-y^2)\left[\frac{dx^2}{x^2-1} + \frac{dy^2}{1-y^2}\right], \quad (2.1)$$

with

$$f = \left(\frac{x-1}{x+1}\right)^{\delta}, \qquad e^{2\gamma} = \left(\frac{x^2-1}{x^2-y^2}\right)^{\delta^2}, \quad (2.2)$$

where σ is a real constant that is used to "control" the physical units of the spatial coordinates. The constant parameter δ lies in the range $-\infty < \delta < +\infty$ with no other restrictions. The degenerate case $\delta = 0$ can be shown to correspond to a flat Minkowski metric. In general, this spacetime describes a static, axisymmetric, vacuum gravitational field. Usually it is interpreted in terms of its multipole moments and corresponds to a non spherically symmetric mass distribution. The parameter δ determines all multipole moments higher than the monopole. In the special case of $\delta = 1$ it reduces to the Schwarzschild metric, as can easily be seen by performing the coordinate transformation $x = -1 + r/m$ and $y = \cos\theta$, and choosing $\sigma = m$, m the Schwarzschild mass. For this reason, in the general case ($\delta \neq 1$) one usually demands that the spatial coordinates lie in the range $x \geq 1$ ($r \geq 2m$) and $-1 \leq y \leq 1$. The hypersurface $x = 1$, ($r = 2m$), represents a true curvature singularity, in accordance with the uniqueness theorems of black holes.

If we extend the Zipoy-Voorhees manifold to $x \geq -1$, then a second curvature singularity appears at $x = -1$. The sector of the manifold contained in the range $-1 \leq x \leq 1$, with $-1 \leq y \leq 1$, can be interpreted as a Gowdy cosmology by means of an appropriate redefinition of the coordinates [13]. This metric is the general solution of the $S^1 \times S^2$ Gowdy equations where ? depends only on time. This shows that the outer ($x = +1$) and inner ($x = -1$) "horizons" are actually surfaces of infinite curvature (naked singularities), but between the "horizons" this metric represents a perfectly viable Gowdy cosmology. Notice that in this case, an apparent singularity appears at the hypersurface $x = y$ which, however, can be removed by means of a suitable coordinate transformation for any integer values of the parameter δ. For values within the range $\delta^2 < 3/2$ with $\delta^2 \neq 1$, the hypersurface $x = y$ corresponds to a true curvature singularity.

There exist certain generalizations of this metric [17] and a study of the geodesic motion in the special case $\delta = 2$ was performed in [18]. Nevertheless, the global properties of the Zipoy-Voorhees spacetime have not been analyzed in detail in the literature except, of course, for the limiting case of the Schwarzschild metric ($\delta = 1$). This is probably due to the fact that only the sector outside the "horizon" ($x \geq 1$) has been considered of physical interest in the context of possible applications for describing the exterior gravitational field of astrophysical objects; however, more general solutions exist in the literature (for a review see, for example, [15]) that are more adequate than the Zipoy-Voorhees metric to study non spherically symmetric mass distributions and their multipole moments and, consequently are more interesting from the point of view of possible astrophysical applications.

Nevertheless, the reinterpretation of a sector inside the "horizon" of the Zipoy-Voorhees manifold as a cosmological Gowdy model [13] and also as a regular S-brane solution, could focus new attention on this metric.

2.2 The regular S0-brane

In this subsection we derive a special solution for a non-twisting regular S-brane by applying an analytical continuation of the Zipoy-Voorhees metric from "inside the horizon". The most general solution that can be obtained in this manner will be presented in section 5. Here we restrict ourselves to the special case where $\delta = 1$. As mentioned before, this limiting case corresponds to the Schwarzschild metric with $x = -1 + r/m$, $y = \cos\theta$, and $\sigma = m$. By applying an analytical i-rotation given by

$$t \to ir\,, \qquad r \to it\,, \qquad \theta \to i\theta\,, \qquad m \to im\,, \tag{2.3}$$

this black hole solution has been used previously in [16] to generate the well-known 4-dimensional S0-brane solution

$$ds^2 = -\left(1 - \frac{2m}{t}\right)^{-1} dt^2 + \left(1 - \frac{2m}{t}\right) dr^2 + t^2(d\theta^2 + \sinh^2\theta\, d\varphi^2)\,. \tag{2.4}$$

The disadvantage of this solution is that it possesses a time-like naked singularity. Therefore it is not appropriate for describing the formation and decay of unstable branes which is expected to be a smooth process free of singularities.

In the terminology of the horizon method, the i-rotation (2.3) can be interpreted as an analytical continuation from "outside the horizon". What we will show now is that in order to avoid the appearance of singularity it is sufficient to apply an i-rotation from "inside the horizon".

Due to the signature change, between the horizons, $x^2 < 1$, the coordinate x becomes timelike and t becomes spacelike. Let us define the coordinates

$$x = \cos\tau\,, \qquad t = r \tag{2.5}$$

for the cosmological sector of this manifold. Then, from eq. (2.1) we obtain

$$ds^2 = \frac{1-\cos\tau}{1+\cos\tau}dr^2 + \sigma^2(1+\cos\tau)^2(1-y^2)d\varphi^2 + \sigma^2(1+\cos\tau)^2\left(\frac{dy^2}{1-y^2} - d\tau^2\right). \tag{2.6}$$

After an appropriate coordinate transformation [13], this metric can be interpreted as a 4-dimensional Gowdy cosmological model characterized by a big bang singularity at $\cos\tau = -1$. We now perform the following i-rotation in the cosmological sector of the Zipoy-Voorhees metric (2.6):

$$\tau \to i\theta\,, \qquad y \to i\frac{\tau}{\sigma}\,, \qquad r \to ir\,. \tag{2.7}$$

This analytical continuation leads to the following metric

$$ds^2 = -\frac{\sigma^2(\cosh\theta+1)^2}{\tau^2+\sigma^2}d\tau^2 + \frac{\cosh\theta - 1}{\cosh\theta+1}dr^2 + (\cosh\theta+1)^2\left[\sigma^2 d\theta^2 + (\tau^2+\sigma^2)d\varphi^2\right]. \tag{2.8}$$

As we discuss in the Conclusions, this process does not guarantee that the resultant metric is an axisymmetric time and angular dependent solution of the vacuum Einstein equations, but one can show that the metric of eq. (2.8) is indeed a solution. An analysis of the corresponding curvature shows that it is asymptotically flat. Moreover, the Kretschmann scalar

$$K = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = \frac{48}{\sigma^4(\cosh\theta + 1)^6} \tag{2.9}$$

is perfectly well defined for all values of θ and does not depend on the time coordinate τ. One important point about this solution is that it does not require a "twist" in space in order to avoid the singularity, as has been demanded in previous regular solutions [6]. This property makes this metric the simplest possible regular S-brane solution. From now on, we will refer to the metric (2.8) as the *regular S0-brane solution*. Generalizations of this solution will be presented in section 5.

3. Global structure

In this section we investigate the main properties of the regular S0-brane derived above. In particular, we are interested in the analysis of its isometries, asymptotic behavior and the underlying R-symmetry.

3.1 Symmetries

Since the spacetime metric (2.8) does not depend explicitly on the spatial coordinates φ and r there exist two Killing vector fields $K_I = \partial_\varphi$ and $K_{II} = \partial_r$. K_I describes the axial symmetry and its norm $|K_I| = (\cosh\theta + 1)^2(\sigma^2 + \tau^2)$ is regular on the entire manifold. The norm of the second Killing vector

$$|K_{II}| = \frac{\cosh\theta - 1}{\cosh\theta + 1} \tag{3.1}$$

vanishes for $\theta = 0$. This shows the existence of a Killing horizon at this hypersurface. Outside this horizon, the norm of K_{II} is positive definite, indicating that the coordinate r is spacelike at all points of the manifold outside the horizon. Notice, however, that if we preserve the additional parameter $\mu > 1$ (see section 5) in the case $\delta = 1$, the norm of this Killing vector $K_{II} = (\mu\cosh\theta - 1)/(\mu\cosh\theta + 1)$ is positive definite on the entire spacetime manifold. In this case no horizon is present. Thus, the parameter μ can be used to eliminate the horizon.

The metric (2.8) is invariant under the discrete symmetry $\theta \to -\theta$, indicating that it is symmetric with respect to the hypersurface $\theta = 0$. None of the metric functions change their sign when passing through the Killing horizon $\theta = 0$ so that no closed timelike geodesics exist and no Cauchy horizon appears. An observer traveling along the spatial coordinate θ is not affected by the presence of the Killing horizon at $\theta = 0$.

3.2 R-symmetry

The coordinate τ in (2.8) is always timelike. The explicit dependence on the spacelike coordinate θ indicates that the metric can represent a spatially localized source. The S-brane worldvolume lies along the spatial direction r so that φ and θ are spatial directions

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transverse to the worldvolume. While in the singular S0-brane solution (2.4) and in the regular Kerr S-brane solution [6, 7] both transverse directions θ and φ contain factors which depend explicitly on time. In the case of the regular S0-brane presented here only the φ direction has cosmological expansion. This fact represents the main difference in the topology of the previous known (singular and regular) S-branes solutions and the regular S0-brane solution (2.8).

The original singular S0-brane solution corresponds to a 4-dimensional manifold of the form $R^{1,1} \times H^2$, where H^2 is a hyperbolic space that determines the R-symmetry. Therefore, it possesses a SO(1, 2) R-symmetry. In the case of the regular Kerr S-brane solution, the 4-dimensional manifold can be identified as a globally non trivial fiber bundle with fiber H^2 over the base space $R^{1,1}$. The twisting in two of the spatial directions, which is the cause of the elimination of the singularity in this spacetime, induces a non trivial global topology in the fiber bundle that corresponds to $R^{1,1} \ltimes H^2$, and the R-symmetry reduces to SO(2) [6].

In the regular S0-brane solution (2.8), the metric of the transverse directions φ and θ becomes

$$ds^2_{\text{R-sym}} = (\cosh\theta + 1)^2(\sigma^2 d\theta^2 + \tilde{\sigma}^2 d\varphi^2)\,, \tag{3.2}$$

where $\tilde{\sigma}$ is a real constant. This is a conformally flat 2-dimensional euclidean space which, in general, is invariant with respect to transformations belonging to the group $S0(1,3)$. This corresponds to the complete lorentzian symmetry and is therefore the most general R-symmetry of all known regular solutions.

3.3 Asymptotic behavior

The asymptotic behavior of the regular S0-brane (2.8) is determined by the behavior of the metric as $\theta \to \infty$ and as $\tau \to \infty$. Let us first consider the spatial asymptote. The only function to be considered is $\cosh\theta$ which behaves as $\exp(\theta)/2$ for $\theta \to \infty$. The spatial asymptote is then given by

$$ds^2_{\theta\to\infty} = \frac{e^{2\theta}}{4}\left[-\frac{\sigma^2 d\tau^2}{\tau^2+\sigma^2} + \sigma^2 d\theta^2 + (\tau^2+\sigma^2)d\varphi^2\right] + dr^2\,. \tag{3.3}$$

If we consider, in addition, values of the constant σ such that $\sigma^2 \ll \tau^2$, this metric reduces to

$$ds^2_{\theta\to\infty} = -\xi^2 dt^2 + d\xi^2 + \frac{\tau_0^2}{\sigma^2}\xi^2 e^{2t} + dr^2\,, \tag{3.4}$$

where we have introduced the new timelike coordinate $t = \ln(\tau/\tau_0)$ and the spacelike coordinate $\xi = (\sigma/2)\exp(\theta)$. This metric corresponds to a Rindler space with an exponential expansion in the direction of the angle φ. This coincides exactly with the spatial behavior of the regular Kerr S-brane solution [6, 7]. In the general case of metric (3.3) it can easily be shown that it corresponds to a Rindler space with an expansion in the angular direction given by a hyperbolic cosine function of time.

We now analyze the temporal asymptote. In the asymptotic limit $\tau \to \infty$ we obtain

$$ds^2_{\tau\to\infty} = (\cosh\theta + 1)^2(-\sigma^2 dt^2 + \sigma^2 d\theta^2 + \tau_0^2 e^{2t} d\varphi^2) + \frac{\cosh\theta - 1}{\cosh\theta + 1} dr^2\,, \tag{3.5}$$

where the time coordinate is given as before by $t = \ln(\tau/\tau_0)$. In the special case $r =$ const., this corresponds to a conformal 2-dimensional Minkowski spacetime with an exponential expansion in the angular direction φ.

Although in section 3.1 we have shown that the Killing horizon at $\theta = 0$ can be removed by means of the free parameter $\mu > 1$, it is interesting to investigate the limiting case ($\mu = 1$) in which the horizon is present. From eq. (2.8) we can obtain the 3-dimensional metric in the near horizon limit which can be written as

$$ds^2_{nhl} = 4\sigma^2(-dT^2 + d\theta^2 + \cosh^2 T\, d\varphi^2) + \frac{1}{4}(e^\theta - 1)^2 dr^2\,, \tag{3.6}$$

where we have introduced a new timelike coordinate by means of $\tau = \sigma \sinh T$. This shows that near the horizon the regular S0-brane behaves as de Sitter space with a time expansion in the angular direction φ. This resembles the behavior of the regular Kerr S-brane, but in the present case crossing the horizon does not imply a change in the signature of the metric so that all the coordinates remain well-behaved.

4. A dilatonic generalization

In this section we will generalize the regular S0-brane solution to include the dilaton field which arises in the low-energy limit of IIA string theory. We first apply a Harrison transformation to the Gowdy cosmological model presented in section 2.2. As a result we obtain an electrovacuum Gowdy solution. The analytic continuation of this solution is then interpreted as describing a regular charged S-brane configuration. Then we make use of a particular symmetry property of the field equations, which follow from the low-energy action of string theory, to generate the dilatonic generalization. We briefly discuss the main properties of this generalized, regular S0-brane solution.

4.1 A charged regular S0-brane

In this section we derive the charged generalization of the regular S0-brane presented in 2.2. The first step consists in deriving a solution of the Einstein-Maxwell field equations which contains the Zipoy-Voorhees metric (with $\delta = 1$) in the limiting case of vanishing electromagnetic field. Clearly, there is in principle an infinite number of possible generalizations of a vacuum solution which include an electromagnetic field, each one corresponding to different sets of charge multipole moments. Here we will present the simplest generalization in which only an additional charge monopole moment is included. In fact, this case has been analyzed by Harrison [20] who proposed a transformation which generates electrovacuum solutions from vacuum ones in the following manner. Let f_0 and γ_0 represent a vacuum solution with metric (2.1), and f and γ represent the corresponding electrovacuum solution with only a charge monopole moment. Then, these two solutions are related by (see, for instance, [21] for details of computations):

$$f = 4f_0[1 + f_0 + \eta_0(1 - f_0)]^{-2}\,, \qquad \gamma = \gamma_0\,, \qquad \eta_0 = (1 - e^2)^{-1/2}\,, \tag{4.1}$$

where e is a real constant which is interpreted as the specific charge, i.e., the ratio between the net charge and the total mass of the source. With this transformation one can easily derive the charged generalization of the Zipoy-Voorhees solution. In the region contained within the "horizons", $-1 \le x \le 1$, we proceed as in section 2.2 and introduce the coordinates $t = r$ and $x = \cos\tau$ to obtain the metric

$$ds^2 = \frac{\sin^2\tau}{(\eta_0 + \cos\tau)^2}dr^2 + \sigma^2(\eta_0 + \cos\tau)^2\left[(1 - y^2)d\varphi^2 + \frac{dy^2}{1 - y^2} - d\tau^2\right], \tag{4.2}$$

and the electromagnetic potential

$$A = -\frac{e\eta_0}{\eta_0 + \cos\tau}dr\,. \tag{4.3}$$

From here we recover the metric (2.6) in the limiting case $e = 0$. Consequently, we can interpret this solution as describing a Gowdy cosmology endowed with a specific electric charge e. We now apply the i-rotation (2.7) on the solution (4.2) and introduce the angle coordinate θ. To obtain a real Maxwell potential we also need to i-rotate the specific charge, i.e. we change $e \to ie$. The resulting metric can be written in the following form:

$$ds^2 = -\frac{\sigma^2(\eta + \cosh\theta)^2}{\tau^2 + \sigma^2}d\tau^2 + \frac{\sinh^2\theta}{(\eta + \cosh\theta)^2}dr^2 + (\eta + \cosh\theta)^2\left[\sigma^2 d\theta^2 + (\tau^2 + \sigma^2)d\varphi^2\right], \tag{4.4}$$

with the potential 1-form,

$$A = \frac{e\eta}{\eta + \cosh\theta}dr\,, \qquad \eta = (1 + e^2)^{-1/2}\,. \tag{4.5}$$

In the limiting case $e = 0$, this metric reduces to the regular S0-brane presented in section 2.2. An analysis of the corresponding curvature shows that the regularity property is not affected by the presence of the additional charge. As one might expect, the constant e determines the charge of the S0-brane.

4.2 The dilatonic field

In this section we generalize the charged regular S0-brane solution to include a dilatonic field. Let us consider the Einstein-Maxwell-dilaton action

$$S = -\int d^4x\,\sqrt{-g}[-R + 2(\triangle\phi)^2 + e^{-2\alpha\phi}F^2]\,, \tag{4.6}$$

where ϕ is the dilatonic field and α is the dilatonic coupling constant which determines the special cases of the theories contained in (4.6). Indeed, if $\alpha = \sqrt{3}$ one obtains the Kaluza-Klein field equations which result from the dimensional reduction of the 5-dimensional Einstein vacuum field equations. In the special case $\alpha = 1$, the action (4.6) coincides with the low-energy limit of string theory with vanishing dilaton potential. It turns out that

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if we restrict ourselves to spacetimes with two commuting Killing vector fields, the field equations following from the variation of (4.6) possess certain symmetry properties which are very helpful in the search for new solutions. In particular, one particular symmetry leads to a transformation that allows us to generate dilatonic solutions from static electrovacuum solutions. This specific transformation can be formulated in the following way. Let

$$f_0\,, \qquad \gamma_0\,, \qquad \phi_0 = 0\,, \qquad F_0 = dA_0 \tag{4.7}$$

represent a particular solution of the field equations following from the action (4.6) with line element (2.1). Then, a simple dilatonic generalization of (4.7) can be obtained by means of the transformation:

$$f = (f_0)^{\frac{1}{1+\alpha^2}}\,, \qquad \gamma = \frac{\gamma_0}{1+\alpha^2}\,, \qquad A = \frac{A_0}{\sqrt{1+\alpha^2}}\,, \qquad e^{2\phi} = (f_0)^{\frac{\alpha}{1+\alpha^2}}\,. \tag{4.8}$$

This transformation can easily be generalized to the case of time and angular dependent charged solutions with no twist such as the solution of section 4.1. The proof essentially resembles the procedure we have used in section 2.2 to derive the regular S0-brane solution. The final result is a new solution of the form:

$$\begin{aligned} ds^2 =& -(\eta+\cosh\theta)^{2\beta}\left(\cosh\theta+\frac{\tau^2}{\sigma^2}\right)^{\alpha^2\beta}\frac{\sigma^2 d\tau^2}{\tau^2+\sigma^2}+\left(\frac{\sinh\theta}{\eta+\cosh\theta}\right)^{2\beta}dr^2+ \\ &+(\eta+\cosh\theta)^{2\beta}\left[\sigma^2\left(\cosh^2\theta+\frac{\tau^2}{\sigma^2}\right)^{\alpha^2\beta}d\theta^2+(\sinh\theta)^{2\alpha^2\beta}(\tau^2+\sigma^2)d\varphi^2\right], \end{aligned} \tag{4.9}$$

$$e^{2\phi} = \left(\frac{\sinh\theta}{\eta+\cosh\theta}\right)^{2\alpha\beta}\,, \qquad \beta = \frac{1}{1+\alpha^2}\,, \tag{4.10}$$

$$A = \frac{e\eta}{\sqrt{\beta}(\eta+\cosh\theta)}\,dr\,. \tag{4.11}$$

This solution is regular for all values of θ and τ. There is no change in the signature of the metric when crossing the Killing horizon situated at $\theta = 0$. It can be considered as the low-energy limit of a solution of IIA string theory (in the Einstein frame). It is therefore possible to construct the corresponding exact S-brane solution in string theory for which one would expect it to describe the decay of an unstable D-brane.

It should be mentioned that the inclusion of the dilatonic field modifies some of the global properties of the original regular S0-brane solution. Unlike the regular S0-brane solution in which only the transverse direction φ changes in time, the dilatonic field introduces an additional time dependence along the spatial direction θ. For the sake of concreteness, we consider now the special case $\alpha = 1$ of the S-brane solution (4.9)-(4.11). At spatial infinity the asymptotic behavior of the metric remains unchanged and is described by the Rindler space (3.3), the electric field vanishes, and the dilatonic field approaches a constant value. Differences appear in the temporal asymptote which is now given by

$$ds^2_{\tau\to\infty} = \sigma^2(\eta+\cosh\theta)(-4d\tilde{\tau}^2+\tilde{\tau}^2\,d\theta^2+\tilde{\tau}^4\,\sinh\theta\,d\varphi^2)+\frac{\sinh\theta}{\eta+\cosh\theta}\,dr^2\,, \tag{4.12}$$

where the new time coordinate $\tilde{\tau}$ is defined by $\tilde{\tau}^2 = \tau/\sigma$. Both spatial directions θ and φ present an asymptotic power expansion, which is different from the asymptotic exponential expansion in the φ-direction of the regular S0-brane solution.

Another interesting feature of this solution is that the dilatonic field also modifies the R-symmetry of the original S0-brane solution. Indeed, for the special case $\alpha = 1$ it is easy to see that the hypersurface transverse to the worldvolume of the brane is described by the metric

$$ds^2_{\text{R-sym}} = \sigma^2(\eta + \cosh\theta)\left[\sqrt{\cosh^2\theta + \tau_0^2}\; d\theta^2 + \sinh\theta\, d\tilde{\varphi}^2\right], \tag{4.13}$$

where $\tau_0 = \tau/\sigma$ is a constant and $\tilde{\varphi} = \sqrt{1+\tau_0^2}\,\varphi$. Unlike the case of the S0-brane solution, this metric is not conformally flat. In fact, without the contribution of the conformal factor $(\cosh\theta + \eta)$ the curvature scalar of the metric (4.13) is given by

$$R = \frac{(\tau_0^2 - 1)\cosh^2\theta - 2\tau_0^2}{2\sigma^2 \sinh^2\theta\,(\cosh^2\theta + \tau_0^2)^{5/2}}\,, \tag{4.14}$$

which vanishes (as well as the curvature components) only asymptotically at spatial infinity. Since it is possible to introduce a local two-bein for the metric (4.13) such that the local metric is euclidean, we conclude that underlying symmetry is SO(2). This means that the lorentzian R-symmetry of the original, regular S0-brane solution becomes reduced to an SO(2) R-symmetry by the presence of the dilatonic field.

To conclude this section we would like to mention that it is possible to consider more general electromagnetic and dilatonic fields. The Harrison transformation makes it possible to generate simple charged generalizations from static vacuum solutions. This simple generalization determines in turn the corresponding dilatonic field according to the transformation (4.8). In fact, all the fields are determined by the seed function f_0 of the static vacuum solution. However, as has been shown in [22], the field equations which follow from the Einstein-Maxwell-dilaton action (4.6) possess more general symmetry properties which can be used to generate electromagnetic and dilatonic fields in terms of an arbitrary harmonic function. This harmonic function is completely independent of the value of the seed function f_0. This means that for the regular S0-brane solution presented in section 2.2 (or its generalization of the next section) we can generate different dilatonic fields with completely different local and global properties. This opens the possibility of searching for regular S-brane solutions with dilatonic fields with any desired properties.

5. A family of non-twisting S-branes

The Zipoy-Voorhees metric (2.1) is valid for any real values of the parameter δ. In this section we generalize the special, regular S0-brane solution of section 2.2 to include the case of an arbitrary value of the parameter δ.

Applying an i-rotation "between the horizons" of the metric (2.1) and choosing the coordinates in a way similar to that of section 2.2, it can be shown that the resulting solution can be written as

$$ds^2 = \sigma^2 f^{-1} e^{2\gamma}\left(\mu^2\cosh^2\theta + \frac{\tau^2}{\sigma^2}\right)\left[-\frac{d\tau^2}{\tau^2+\sigma^2} + \frac{\mu^2\sinh^2\theta\; d\theta^2}{\mu^2\cosh^2\theta - 1}\right] +$$

$$+fdr^2 + f^{-1}(\mu^2\cosh^2\theta - 1)(\tau^2+\sigma^2)d\varphi^2 \ , \tag{5.1}$$

with

$$f = \left(\frac{\mu\cosh\theta - 1}{\mu\cosh\theta+1}\right)^{\delta} \ , \qquad e^{2\gamma} = \left(\frac{\mu^2\cosh^2\theta - 1}{\mu^2\cosh^2\theta + \tau^2/\sigma^2}\right)^{\delta^2} \ , \tag{5.2}$$

where μ is an arbitrary real constant in the range $1 < \mu^2 < \infty$. This is a 4-dimensional axisymmetric, time and angular dependent solution of Einstein's vacuum field equations. It is regular on the entire manifold as can be seen from the Kretschmann scalar

$$K = \frac{16\delta^2}{\sigma^4}\frac{(\mu\cosh\theta-1)^{-2\delta^2+2\delta-2}}{(\mu\cosh\theta+1)^{2\delta^2+2\delta+2}}\left(\mu^2\cosh^2\theta+\frac{\tau^2}{\sigma^2}\right)^{2\delta^2-3}L(\tau,\theta)\,, \tag{5.3}$$

where

$$L(\tau,\theta) = 3(\mu\cosh\theta-\delta)^2\left(\mu^2\cosh^2\theta+\frac{\tau^2}{\sigma^2}\right)+(\delta^2-1)\left(1+\frac{\tau^2}{\sigma^2}\right)[\delta^2-1+3\mu\cosh\theta(\mu\cosh\theta-\delta)]\,. \tag{5.4}$$

For $\mu^2 > 1$ and any values of the parameter δ this scalar does not diverge at any point of the manifold. The solution is also asymptotically flat. For $\delta = 1$ we recover the regular S0-brane solution discussed in section 2.2. Only in this special case, the parameter μ can take the degenerate value $\mu = 1$. However, one could also keep this parameter positive and $\mu > 1$ and use it to eliminate the Killing horizon which appears along the spatial coordinate θ (see section 3).

It is clear that the solution (5.1) admits a much richer structure than the regular S0-brane solution (2.8). For instance, whereas the curvature of the regular S0-brane does not depend explicitly on time, the curvature of the generalized solution is always time-dependent, and this dependence can be changed arbitrarily by means of the parameter δ. This indicates that the global properties will also depend on the parameter δ.

The generalization of this family of regular solutions to include an electromagnetic field and a dilatonic field is straightforward. The symmetry properties mentioned in section 4 are valid and can be applied in this general case also.

6. Conclusions

We have derived a family of 4-dimensional regular S-brane solutions that solve the singularity problem of S-branes without requiring a twist in space. The simplest representative of this family of solutions is obtained by applying the method of analytic continuation to the Schwarzschild spacetime, but instead of applying it from "outside the horizon", we first "cross" the horizon where the spacetime can be interpreted as a Gowdy cosmology and then perform the i-rotation. This shows that the analytical continuation of the Schwarzschild spacetime "inside the horizon" can get rid of the singularity without introducing a twist in space. In the general case, the family of metrics can be interpreted as a Zipoy-Voorhees regular S-brane solution.

It is important to clarify a point concerning the method used here and in other works to derive this type of new spacetimes. Modern solution generating techniques were developed almost thirty years ago and have been extensively used to generate solutions of

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the Einstein-Maxwell field equations with two Killing vector fields. Simple examples of these methods are the Harrison transformation [20] as given in eq. (4.1) and the dilatonic transformation (4.8). All these methods are based upon the existence of certain continuous deformations (Lie transformations) of the field equations which are the elements of an infinite dimensional group of transformations, first discovered by Geroch [23] in the stationary, axisymmetric Einstein vacuum field equations. The study of the Geroch group gave rise to the development of well established methods that allow us to generate new solutions from a known seed solution. It is clear that the "horizon method", the i-rotation and other similar methods cannot be included within the category of continuous deformations of the field equations and should be considered at most as "tricks" that incidentally happen to generate new solutions. Consequently, one can not assure a priori that the application of these "tricks" really lead to new solutions. One always should test the resulting metrics with the corresponding field equations. In fact, in our experience we have found cases in which these "tricks" fail to work. Probably, the reason why they sometimes happen to lead to new solutions is related to the existence of yet unknown discrete symmetries of the field equations.

The S-branes solutions presented here correspond to asymptotically flat, time and angular dependent backgrounds. They depend on a real parameter δ, which could be used to "control" the time and angular dependence of the corresponding spacetime metric and curvature, and on an additional parameter μ, which in the limiting case $\delta = 1$ can be used to remove the Killing horizon that appears along one of the spatial coordinates.

In this work we have analyzed in detail the global properties of only the simplest regular S0-brane, i.e., when $\delta = \mu = 1$. This case is relatively simple because the curvature does not depend explicitly on time. Although this regular solution does not have a twist in space, its asymptotic behavior resembles to some extent that of the twisting Kerr S-brane. We have seen that the R-symmetry of the regular S0-brane solution corresponds to that of a 2-dimensional conformally flat space, in contrast to the hyperbolic space R-symmetry of the singular S0-brane. This kind of symmetry reduction has been proposed earlier as a possible approach to avoid the singularity, and has been used recently to derive regular twisting S-brane solutions. Our results show that non-twisting, simpler, regular solutions can be obtained when the R-symmetry corresponds to the general lorentzian symmetry. However, when we consider the additional dilatonic field the R-symmetry becomes SO(2) as in the case of the twisting Kerr S-brane. This is again in the spirit of the idea that the reduction of the R-symmetry allows us to overcome the singularity problem of S-branes. Our analysis of the properties of the regular S0-brane solution and its dilatonic generalization shows that it can be used to describe the formation and decay of an unstable D-brane.

The family of regular S-brane solutions derived in section 5 offers several possibilities to continue the investigation of regular S-brane configurations. For instance, it would be interesting to study the case $\delta = 2$ and $\mu^2 > 1$ for which one can show that the curvature depends explicitly on time, unlike the regular S0-brane solution whose curvature depends only on one spatial coordinate. Also, the corresponding metric contains an explicit time and angular dependence in both spatial directions θ and φ which are the directions transverse to the worldvolume of the brane. The asymptotic behavior of the corresponding S-brane

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solution and its dilatonic generalization will present different possible scenarios that could be of interest, especially in the context of the formation and decay of more general unstable branes. This task is currently under investigation.

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